**Scattering**

Let’s do a few scattering scenarios.

**Compton Scattering**

We touched on particle dynamics when we discussed the Abraham-Lorenz formula for the radiative back-reaction force back in the EM folder. I’ve gotta do Compton scattering somewhere, so I guess I’ll do it here. General setup is we are sending an X-ray into a stationary electron target (graphite) and observing how the wavelength changes as a function of its deflection angle. Caveat: the electrons in the graphite target are clearly not stationary in any sense, but equal numbers will be going one way vs. the opposite way, and so I think we can just imagine its stationary for simplicity, as opposed to assuming some initial velocity for the electron and then averaging our result over all possible velocities.

Diagram, schematic

Description automatically generated

So let the X-ray be going to the right with momentum ℏ**k**, and energy ℏω (ℏ = h/2π, and k = 2π/λ, ω = 2πf). Note that since fλ = c, necessarily, the energy is not independent of the momentum. And let the electron be initially stationary with momentum **p** = 0, and energy E = mc2, but then emerge from the collision with momentum **p´** = m**v**γ, and energy E´ = mc2γ. The photon will emerge from the collision with momentum ℏ**k´**, and energy ℏω´. Then conservation of momentum and energy require,



So **k**´ and **p**´ are independent variables, which gives us 6 total unknown d.o.f.. And we have 4 equations. So we cannot predict the final momenta and energies from these equations alone. But if we suppose that we know the scattering angle between **k** and **k**´, that will reduce eliminate 2 d.o.f., and so we can use these equations to solve for k´ with that information. To start, we’ll rearrange and square both sides of the momentum equation,



And now fill this into the energy equation (we’ll square it first)



Now ω = kc, so let’s use that to eliminate ω and ω´,



Can divide both sides by kk´,



which translates to:



This explanation of the angular dependence of the frequency of scattered photons was one of the pieces that proved EM radiation could act as a particle. Well, let’s see if we can do this, w/o presuming the electron is initially at rest. We will presume we know the initial momentum/energy of the electron. And like before, we’ll also presume to know the scattering angle between **k** and **k**´. So starting over,



Can rearrange and square both sides of the momentum equation,



And now fill this into the energy equation (we’ll square it first)



Now ω = kc, so let’s use that to eliminate ω and ω´,



Since we’re presuming to know the direction of **k**´ (i.e., that angle θ above, in the diagram) we really have just one unknown in this equation, its magnitude k´. So we could solve for k´. But at this point, I think I’ll average over all possible initial electron momenta, **p**. But this should be zero, since electrons would be equally likely to go one way vs. the other. But that doesn’t mean its magnitude p would be zero. So then we have:



Can divide both sides by kk´,



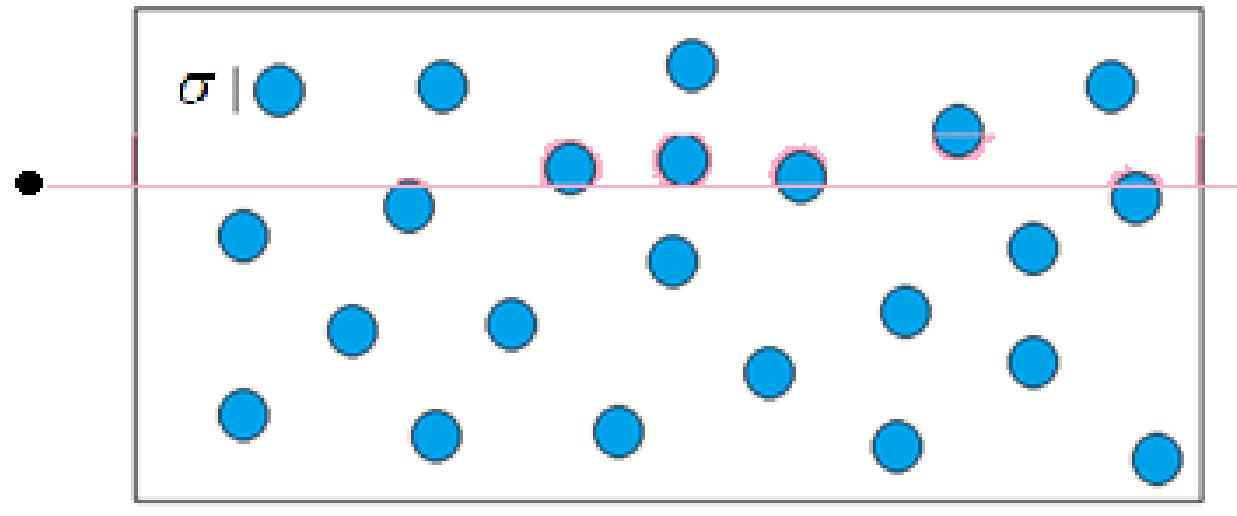
which translates to:



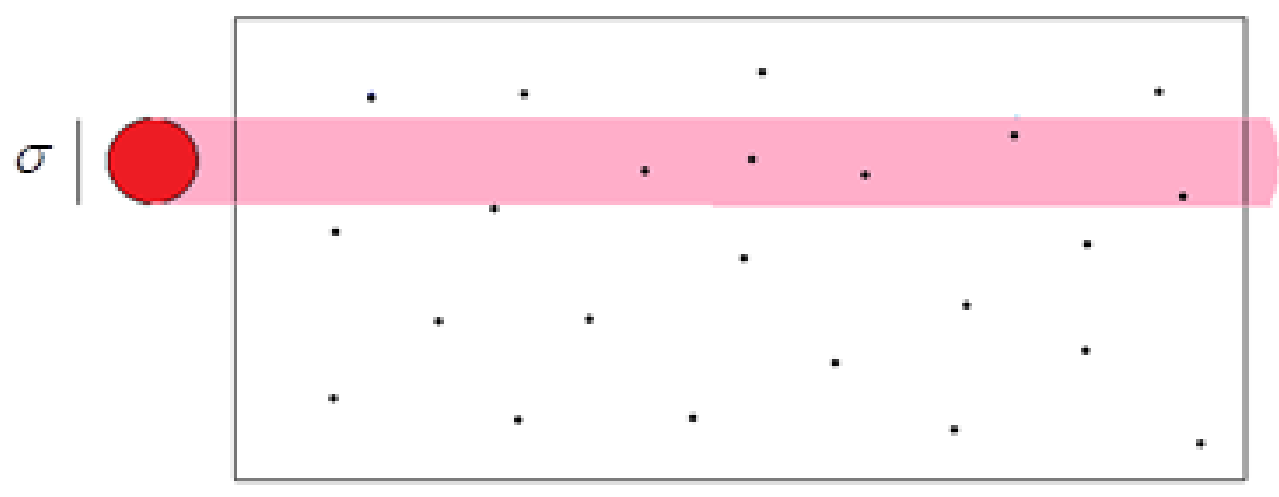
If the rest energy is substantially larger than the kinetic energy, then this reduces to our previous result.

**Nuclear Scattering**

Consider sending a beam of electrons, say, into a plate of copper atoms of a certain thickness, Δx. Given an initial beam intensity, I0, we’d like to know the intensity, I, of the beam emerging to the other side. We can imagine the atoms to have a scattering cross-section, σ, which can be calculated from quantum mechanics. We can imagine an electron in the beam will only emerge to the other side if it doesn’t scatter. So we need to work out the number of particles that will not have scattered. Or we can complimentarily look at the number of particles that have scattered. Consider a slab of thickness dx, and an electron barreling into the copper slab, with Cu atoms arranged in a lattice (doesn’t look like a lattice, sorry). The Cu atoms will have number density n = N/V.



We can consider a complimentary situation where the copper atoms have zero thickness, and it is the electron itself which has cross-section σ.



Then the difference in number of particles that enter vs. emerge from our slab is just the number that have scattered. A single electron will have scattered a number of times (if it could scatter multiple times) equal to the number of Cu atom dots its trajectory intersects. This is the volume of its trajectory, σdx, times the number density of Cu atoms, n. So number of scatterings = σdx∙n. So we can say, for the entire beam, the number of scatterings will be N(x)∙nσdx, where N(x) is the number of electrons on the left side. So we have:



And the solution is:



We can use measurements like this to experimentally work out the scattering cross-section of various atoms, say.

**Example**

An electron beam with an energy 200MeV passes through 50cm of copper. After passing through the copper, the intensity of the beam decreases to 1/5 its original value. What is the nuclear cross-section of copper atoms.

Copper has a density: ρ = 8.96 g/cm3, and an atomic mass of 63.54. So the number density of Cu is:



So,

